
Neutrino Masses and Proton Decay in Minimal Trinification

Sören Wiesenfeldt



Based on work with J. Sayre and S. Willenbrock
Phys. Rev. D73, 035013 (2006) [hep-ph/0601040]

Grand Unification and Proton Decay

dominant decay mode	low-energy Supersymmetry?	
	YES	NO
$p \rightarrow e^+ \pi^0$		
$p \rightarrow \bar{\nu} K^+$		

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Proton decay mediating gauge bosons are absent.

Trinification

[Achiman, Stech 1978; de Rújula, Georgi, Glashow 1984; Babu, He, Pakvasa 1986]

$$G_{\text{TR}} = \text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R \times \mathbb{Z}_3$$

- rank 6;
- \mathbb{Z}_3 guarantees that gauge couplings coincide at M_U ;
- no need for adjoint Higgs fields;
- up to five light Higgs doublets in its minimal version.
 - Gauge-coupling unification may result at $M_U \simeq 10^{14}$ GeV without supersymmetry (depending on the masses of the Higgs bosons and the additional matter).

Minimal Trinification

Gauge group: $G_{\text{TR}} = \text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R \times \mathbb{Z}_3$

Fermions: $(3, \bar{3}, 1) \oplus (\bar{3}, 1, 3) \oplus (1, 3, \bar{3}) \equiv \psi_Q \oplus \psi_{Q^c} \oplus \psi_L$

$$\psi_Q \rightarrow (3, \bar{2}, \tfrac{1}{6}) \oplus (3, 1, -\tfrac{1}{3}),$$

$$\psi_Q = \begin{pmatrix} (-d & u) & B \end{pmatrix}$$

$$\psi_{Q^c} \rightarrow (\bar{3}, 1, -\tfrac{2}{3}) \oplus 2 (\bar{3}, 1, \tfrac{1}{3}),$$

$$\psi_{Q^c} = \begin{pmatrix} \mathcal{D}^c \\ u^c \\ \mathcal{B}^c \end{pmatrix},$$

$$\psi_L \rightarrow (1, 2, \tfrac{1}{2}) \oplus 2 (1, 2, -\tfrac{1}{2}) \oplus (1, 1, 1) \oplus 2 (1, 1, 0),$$

$$\psi_L = \begin{pmatrix} (\mathcal{E}) & (E^c) & (\mathcal{L}) \\ \mathcal{N}_1 & e^c & \mathcal{N}_2 \end{pmatrix}.$$

In addition to the 15 SM fermions, there are **12 new fermions**:

- one vector-like down quark and lepton doublet ($5 + \bar{5}$ of $\text{SU}(5)$);
- one sterile (i.e., $B - L = 0$) neutrino.

Breaking of G_{TR}

Breaking to G_{SM} by a pair of $\Phi_L (1, 3, 3^*) = \begin{pmatrix} (\phi_1) & (\phi_2) & (\phi_3) \\ S_1 & S_2 & S_3 \end{pmatrix}$ with

$$\langle \Phi_L^1 \rangle = \begin{pmatrix} \begin{pmatrix} \\ \\ 0 \end{pmatrix} & \begin{pmatrix} \\ \\ 0 \end{pmatrix} & \begin{pmatrix} 0 \\ 0 \\ v_1 \end{pmatrix} \end{pmatrix} \text{ and } \langle \Phi_L^2 \rangle = \begin{pmatrix} \begin{pmatrix} \\ \\ v_2 \end{pmatrix} & \begin{pmatrix} \\ \\ 0 \end{pmatrix} & \begin{pmatrix} \\ \\ 0 \end{pmatrix} \end{pmatrix}$$

v_1 and v_2 break G_{TR} to $\text{different } \text{SU}(3)_C \times \text{SU}(2)_L \times \text{SU}(2)_R \times \text{U}(1)$

Of the six Higgs doublets, one linear combination is eaten by the gauge bosons that acquire unification-scale masses.

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Yukawa couplings:

$$Y_Q = \psi_{Q^c} \psi_Q (g_1 \Phi_L^1 + g_2 \Phi_L^2),$$

$$\psi_{Q^c} \psi_Q \Phi_L^a \equiv (\psi_{Q^c})_j^i (\psi_Q)_k^j (\Phi_L^a)_i^k$$

$$Y_L = \frac{1}{2} \psi_L \psi_L (h_1 \Phi_L^1 + h_2 \Phi_L^2),$$

$$\psi_L \psi_L \Phi_L^a \equiv \epsilon^{ijk} \epsilon_{rst} (\psi_L)_i^r (\psi_L)_j^s (\Phi_L^a)_k^t$$

$$\rightarrow \text{Heavy states: } B^c = c_\alpha \mathcal{D}^c + s_\alpha \mathcal{B}^c, \quad E = -s_\beta \mathcal{E} + c_\beta \mathcal{L}, \quad \tan \alpha = \frac{g_1 v_1}{g_2 v_2}$$

$$\text{massless: } d^c = -s_\alpha \mathcal{D}^c + c_\alpha \mathcal{B}^c, \quad L = c_\beta \mathcal{E} + s_\beta \mathcal{L}, \quad \tan \beta = \frac{h_1 v_1}{h_2 v_2}$$

$$N_1 = s_\beta \mathcal{N}_1 - c_\beta \mathcal{N}_2, \quad N_2 = -c_\beta \mathcal{N}_1 - s_\beta \mathcal{N}_2$$

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For simplicity, we choose $n_{1,2,3} = 0$ here.

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light fermion masses

No relation between the masses of the quarks and leptons; the minimal model is sufficient to describe the masses of the quarks and charged leptons.

$$m_u = g_1 u_2, \quad m_d \simeq g_1 u_1 s_\alpha,$$

$$m_{\nu, N_1} = h_1 u_2, \quad m_e \simeq h_1 u_1 s_\beta, \quad m_{N_2} \simeq \frac{h_1^2 u_1 u_2 s_\beta}{\sqrt{h_1^2 v_1^2 + h_2^2 v_2^2}}.$$

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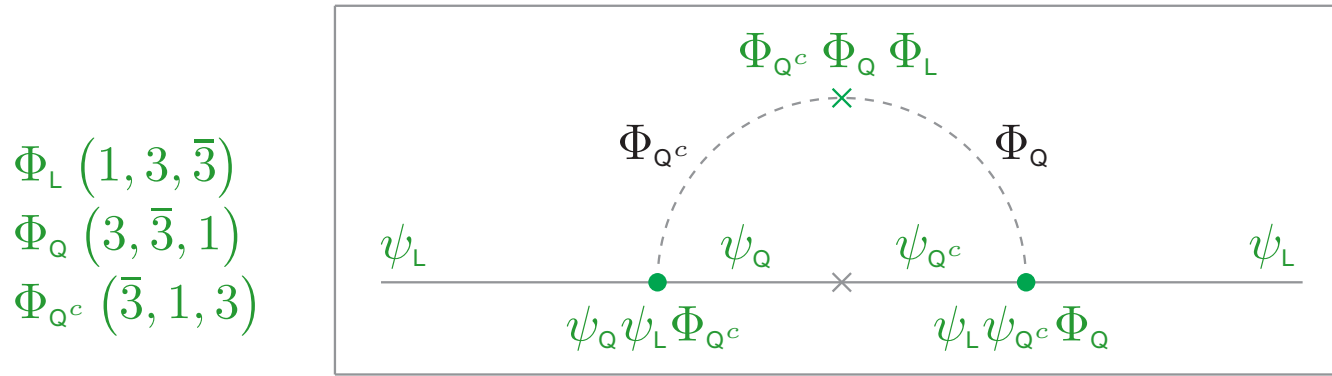
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The **active neutrino**, ν , together with N_1 forms a Dirac state, whereas the **sterile** N_2 receives a small Majorana mass.

Radiative Seesaw Mechanism

One-loop diagrams appear due to the coupling of the neutral fermions to **color-charged Higgs bosons** and the cubic couplings of the Higgs fields.



Yukawa couplings including cyclic permutations,

$$\mathcal{L}_q = g (\psi_{Q^c} \psi_Q \Phi_L + \psi_L \psi_{Q^c} \Phi_Q + \psi_Q \psi_L \Phi_{Q^c}) + \text{h.c.}$$

Higgs potential with quadratic and cubic terms only,

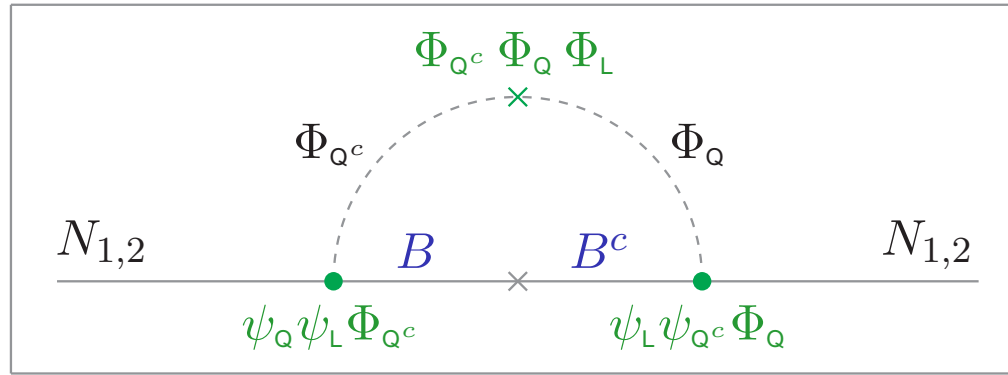
$$\mathcal{L}_h = m^2 (\Phi_Q^* \Phi_Q + \Phi_{Q^c}^* \Phi_{Q^c} + \Phi_L^* \Phi_L) + [\gamma_1 \Phi_{Q^c} \Phi_Q \Phi_L + \gamma_2 (\Phi_L \Phi_L \Phi_L + \text{cyclic}) + \text{h.c.}]$$

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Dominant graphs:

$$\begin{aligned}\Phi_Q & (3, \bar{3}, 1) \\ \Phi_{Q^c} & (\bar{3}, 1, 3)\end{aligned}$$



→ mass matrix for neutrinos (ν, N_1, N_2)

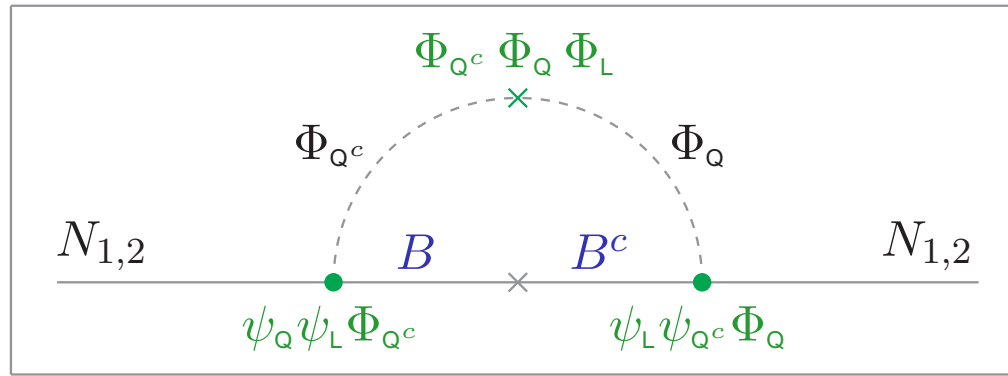
$$M_N \simeq \begin{pmatrix} 0 & -h_1 u_1 & 0 \\ -h_1 u_1 & s_{\alpha-\beta} c_\beta g^2 F_q(B) & (s_{2\beta} s_\alpha - c_\alpha) g^2 F_q(B) \\ 0 & (s_{2\beta} s_\alpha - c_\alpha) g^2 F_q(B) & c_{\alpha-\beta} s_\beta g^2 F_q(B) \end{pmatrix}, \quad F_q(q) \propto m_q \text{ (loop integral)}$$

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This mechanism is absent in models with low-energy supersymmetry.

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sterile neutrinos obtain masses $\lambda_N \sim F_q(B) \sim \mathcal{O}(M_U)$,

active neutrino is light, $\lambda_\nu \sim \frac{(h_1 u_1)^2}{g^2 F_q(B)} \simeq 0.1 \text{ eV}!$

Neutrino Hierarchy

The neutrino hierarchy is related to the couplings in the quark sector.

The dominant 1-loop contributions are those with the heaviest quark, B_3 .

→ three-generational mass matrix for the sterile neutrinos (both N_1 and N_2),
 $M^N \sim (g^{3i} g^{j3} + g^{i3} g^{3j}) F_{B_3}$

Assume a structure like

$$g \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad \epsilon^2 \simeq \frac{m_c}{m_t}. \quad [\text{Lola, Ross 1999}]$$

$$\Rightarrow \quad m_3^N \sim m_2^N \sim F_{B_3} \sim 10^{12} \text{ GeV}, \quad m_1^N \sim \epsilon^4 F_{B_3} \sim 10^8 \text{ GeV}.$$

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Light neutrinos: eigenvalues are proportional to $\frac{h^2}{g^2}$ due to the common loop-integral

→ hierarchy is determined by the hierarchy of h

→ **quasi-degenerate masses or a normal hierarchy.**

Proton Decay

Quarks and leptons in different multiplets. → No proton decay via gauge bosons.

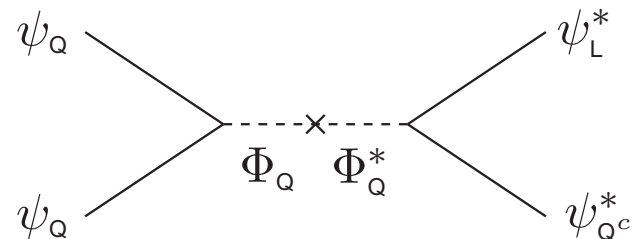
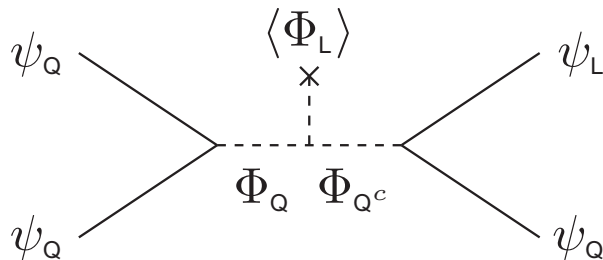
Instead, proton decay is mediated by Φ_{Q^c} and Φ_Q .

These dimension-six operators are suppressed by small Yukawa couplings,

$$\left[(g \hat{s}_\beta) h QQQ L + g \left(-\hat{s}_\alpha^\top h \right) d^c u^c e^c u^c \right] - \left[g^* h Q Q e^{c*} u^{c*} + (g \hat{s}_\beta) \left(-\hat{s}_\alpha^\top h \right)^* d^{c*} u^{c*} Q L \right]$$

$[\hat{s}_\alpha (\hat{s}_\beta)]$: three-generational analogue of the mixing between \mathcal{D}^c and \mathcal{B}^c (\mathcal{E} and \mathcal{L})

→ **Flavor non-diagonal decay dominant**, in particular $p \rightarrow \bar{\nu} K^+$.



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Calculate the decay width using chiral perturbation theory,

$$\Gamma = \left| \sum \mathcal{K}_{\text{had}} C \right|^2, \quad C = \mathcal{C} \frac{1}{\gamma_1^2 v_1^2 - m^4} A \begin{cases} \gamma_1 v_1 \beta & (LLLL, RRRR) \\ -m^2 \alpha & (LLRR, RRLL) \end{cases}, \quad \mathcal{C} = (g h)$$

→ Estimated lifetime: $\tau \simeq \left(\frac{1}{gh} \right)^2 \times 10^{28} \text{ years}$.

Proton Decay

$$\text{Lifetime: } \tau \simeq \left(\frac{1}{g h} \right)^2 \times 10^{28} \text{ years} \quad \Rightarrow \quad g h \lesssim 10^{-3}$$

$$g \sim \begin{pmatrix} \epsilon^4 & \epsilon^3 & \epsilon^3 \\ \epsilon^3 & \epsilon^2 & \epsilon^2 \\ \epsilon & 1 & 1 \end{pmatrix}, \quad \epsilon^2 \simeq \frac{m_c}{m_t},$$

$$\frac{h^{11}}{h^{33}} \sim \frac{m_e}{m_\tau}$$

mode	dominant coeff.	exp. limit [y]
$\bar{\nu} K^+$	$g^{23} h^{11}$	2.2×10^{33}
$\bar{\nu} \pi^+$	$g^{13} h^{11}$	2.5×10^{31}
$\mu^+ K^0$	$g^{12} h^{12}$	1.4×10^{33}

The decay width of $p \rightarrow \bar{\nu} K^+$ is close to the experimental limit.

Supersymmetric Model

In the presence of supersymmetry, unification occurs with two light Higgs doublets (and their superpartners), or even just one in the split supersymmetry scenario.

Neutrinos acquire eV-scale masses only if the mass differences of the SUSY partners is of order M_U ; the lifetime of the gluino restricts the sfermion masses, $m_s \lesssim 10^{14}$ GeV.

[Gambino, Giudice, Slavich 2005]

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The $LLLL$ and $RRRR$ operators mediating proton decay have mass-dimension five, suppressed by $(m_s M_U)^2$. The decay rate is naturally consistent with the experimental limit if the sfermion masses are above a few hundred TeV.

- The model with weak-scale SUSY needs ‘flavor suppression’, similar to models such as SU(5) [cf., e.g., Bajc, Perez, Senjanovic 2002, Emmanuel-Costa, SW 2003];
- proton decay is unobservable in the split-SUSY case.

The mixed operators, $LLRR$ and $RRLR$ arise from D terms, so will not lead to observable decay.

Summary

- The **minimal trinified model**, $G_{\text{TR}} = \text{SU}(3)_C \times \text{SU}(3)_L \times \text{SU}(3)_R \times \mathbb{Z}_3$ is an interesting candidate for **non-supersymmetric unification**.

Breaking is achieved by only two $\Phi_L (1, 3, \bar{3})$ representations which include **five potentially light Higgs doublets**.

- **Sterile Neutrinos** become massive with $M \gg M_{\text{EW}}$ via **radiative seesaw mechanism**; at M_{EW} , only the SM fermions remain.
- No need to introduce intermediate scales, additional Higgs fields, or higher-dimensional operators.
- **Proton decay** is mediated by **color-charged Higgs bosons**.
The decay mode $p \rightarrow \bar{\nu} K^+$ is dominant. (↗ SUSY models with dim-5 ops.)
- **Possibility to verify model**:
 - **no SUSY particles at TeV scale**;
 - **detection of $p \rightarrow \bar{\nu} K^+$ as dominant decay mode**
- Results are also valid for SUSY models, where scalars are as heavy as M_U .
 - Proton decay is unobservable.